

Using Open-ended Tasks to Build Models and Construct Patterns



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discuss the
construction and
pattern work produced
by young children in
response to open-
ended tasks.

Open-ended tasks focused on content specific features are regarded as an effective way to promote particular concept development and to elicit higher-order thinking (Sullivan, Griffioen, Gray, & Powers, 2009). Such tasks may vary widely in their focus and approach and they can be formulated without unnecessary complexity. When mathematics tasks are designed in an open-ended fashion they can also provide flexible opportunities to cater for a range of differing student abilities. In this article it is shown that they can be appropriately utilised in the development of measurement, space and geometry, number, and pattern concepts to achieve a range of learning outcomes.

Current research suggests students' exploration of space and geometry is not sufficiently encouraged in early mathematics classrooms (Casey, Andrews, Schindler, Kersh, Samper, & Copley, 2008). It is recommended that the development of spatial awareness be fostered using block building activities that reflect students "intuitive and informal capabilities" (Clements & Sarama, 2007, p. 139). In a study of preschoolers and kindergarteners, Papic (2007) found that children can represent, symbolise, abstract and generalise by exploring patterns in a variety of ways, including repeating, growing, and spatial forms. A strong awareness of pattern and structure is also thought to provide a basis for understanding space and

geometry concepts (Mulligan, Mitchelmore, Kemp, Marston, & Highfield, 2008). Each of these studies employed the careful design and implementation of open-ended tasks that explicitly linked space and geometry, and pattern and structure.

The Year 1 classroom study

A classroom-based action research study investigated the development and implementation of a series of open-ended tasks focused on two-dimensional and three-dimensional shape, patterning, and spatial awareness through the working mathematically processes central to the NSW Mathematics K–6 syllabus.

The participants comprised one mixed ability class of 19 Year 1 students, aged 6 to 7 years, from a Sydney metropolitan public school, drawn from high socio-economic, white Anglo-Saxon backgrounds. The majority was identified by the classroom teacher as capable of achieving at or beyond Stage 1 mathematics outcomes (Board of Studies NSW, 2002). The classroom teacher acted as participant researcher and is the first author of this paper.

Task description, implementation and feedback

Several tasks were implemented during regular class mathematics time comprising six sessions of one-hour duration spaced throughout a school term. Students were seated individually, located at a distance from others so as to encourage independent thinking and recording of their own responses. This arrangement differed markedly from that used for regular small-group collaborative problem solving and routine mathematics activities.

One of the open-ended problems related to a patterning “tower” task, adapted from the work of Papic (2007), had been completed

in previous lessons. The task presented a realistic problem with the aim of fostering a high level of engagement and allowing full participation for students of all abilities. The task was as follows:

Remember the tower you built last term; it was next to a house. This house showed a pattern too. The house was knocked down and you have been asked to rebuild it. It must show a pattern or several patterns. Imagine what your house looks like. Use the cubes to build it. Draw it. Write about your house. Explain how you made a pattern.

The teacher discussed the students’ earlier explorations of patterning completed in previous lessons. This encouraged them to build on their prior learning by creating and applying simple and complex repetitions in new ways. In this study, students constructed and re-constructed their models and were then required to draw them with the model in view. They were encouraged to think about the pattern structure and spatial features within their models and to explain or justify their observations in writing while the teacher took digital photographs. They described features of their houses including any walls, windows, doors, or any shapes or patterns that were evident and wrote a sentence justifying the pattern: “My house shows a pattern because ...”. Responses were not shared until the final session in order to allow students ample time to reflect on their work and make adjustments to their thinking in consultation with the teacher. The teacher assisted some students who had difficulty scribing their responses.

Collecting and analysing responses

The teacher made notes on her observation of 10 of the 19 students during the lesson and immediately following the lesson. Students’ drawings and their written explanations were collected and analysed for elements of mathematical thinking (representing,

verifying/applying and justifying); and for characteristics of pattern structure, two- and three-dimensional properties, and transformation skills.

General findings

All students successfully made a three-dimensional model depicting the structure of a house rather than a simple tower in vertical formation that they had been limited to in earlier tasks. Sixteen of the 19 students modelled and drew their houses depicting the structure of one or more complex repeating patterns, a border or cyclic pattern. The remaining three students were unable to construct patterns successfully. The 16 students were able to explain and justify the patterns they created and some of the relationships between parts of their model in drawn and/or written form. Several students were able to rotate their structure mentally to view it from other orientations before doing so physically. The following examples represent a range of responses. (Pseudonyms have been used to preserve the students' anonymity).

Individual student responses

Grace built on her prior experience of simple AB repetitions in single towers to construct a rectangular pattern showing an ABCD unit of repeat as a complex three-dimensional structure (Figure 1).

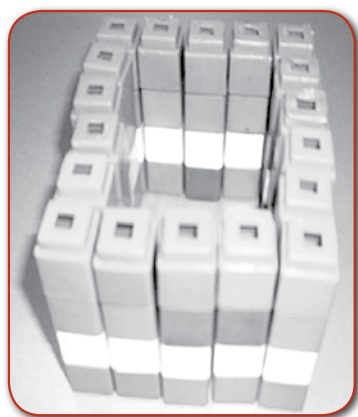


Figure 1. Grace's model using single towers from top to bottom (orange, green, white and blue).

She created the model by aligning a total of 18 towers in a border style rectangular pattern recognising the ABCD repetition by stating: "It goes orange, white, green, blue each time". She incorrectly counted the number of towers because she did not realise that the corner towers need not be counted twice. She said, "It has 22 blues... 22 greens... 22 whites... 22 oranges...". Grace used multiplicative thinking to describe numerical features of her pattern. For example, she said, "There are four sides, there are four in each row, you count by fours... it has 20 rows." Her use of the term "row" was meant to be "column". Grace's drawing (Figure 2) shows a complex three-dimensional configuration in two dimensions using the unit of repeat (even though she misaligns the number of units on each side of the model). She also makes visual transformations by drawing a "flattened" perspective.

Evan's model, shown in Figure 3, depicts a border as a second layer showing an ABBA repetition. He explained: "It shows a pattern because red, green, green, red and this is another way, blue, red, red, green and I know what comes next... blue at the corners". Thus he identified another unit of repeat vertically as "blue, red, red, green." Evan depicted both horizontal and vertical patterns in the structure and made it clear that he understood the different units of repeat and the row and column structure.

Evan constructed four identical faces by making the model in segments ("walls"). The model also shows a

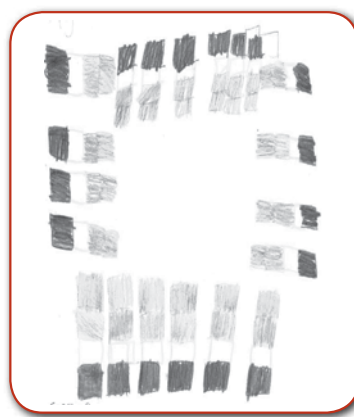


Figure 2. Grace's drawing of her model.



Figure 3. Evan's model of a house with corners built top to bottom as blue, red, red green.

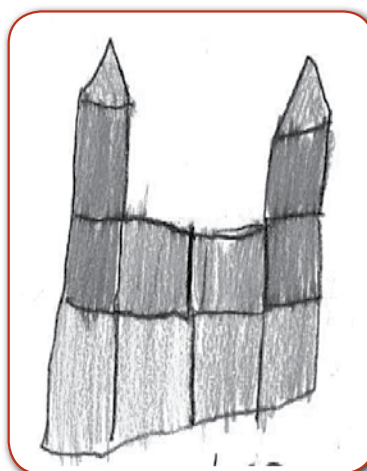


Figure 4. Evan's drawing of the front view of the model.

symmetrical pattern. He said that his house “is shaped like a cube”, recognising the approximately equal-sized length, width and height, if the “chimney/turret” line is visualised. His drawing showed how he visualised the faces of the house by replicating the same pattern on each side (Figure 4). Evan's representation showed that he could reflect more deeply about common features of the structure, such as noticing that the walls must be congruent.

Andrew created an unusual model of a house, shown in Figure 5. It comprised two identical layers, each one approximately circular and using an ABB pattern with a triangular centre (A as green, and B). His model also showed an integrated and complex unit of repeat in another way: an AB pattern using colour; green (two blocks) and blue (one block).

Andrew showed consistency in the application of the pattern structure by replicating it and aligning it as a second “layer”.



Figure 5. Andrew's model of a house.



Figure 6. Andrew's drawing of his model depicting a circular pattern.

Figure 6 shows the ABB unit of repeat represented more times than the model, and he drew the second layer as a concentric pattern. He attempted to represent the model accurately using the centre and two layers, and may have focused on ensuring his cubes were repeating in a circular motion rather than being accurately joined without gaps. When questioned about the spatial features, he stated that he saw a variety of two-dimensional shapes including triangles and a “diamond” [rhombus] and the idea of a centre: “My house has a triangle hole in the middle.”

Dylan's model, shown in Figure 7, was a rectangular prism that was hierarchically complex. He made the model by constructing four individual towers, and then connecting the towers to create a series of bridges. He showed an awareness of pattern and structure by aligning the vertical and horizontal frames on each side of the model. This demonstrated flexibility in his thinking and an ability to see relationships between the parts.



Figure 7. Dylan's model of a house (top to bottom as orange, yellow alternating cubes and yellow in the middle column).

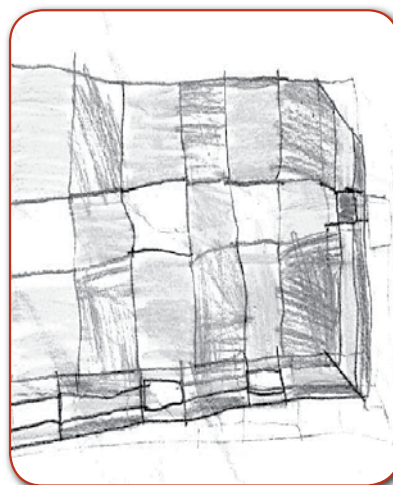


Figure 8. Dylan's drawing of his model.

Dylan's model showed a simple AB repetition as "yellow, orange, yellow, orange..." vertically. He also identified a letter-shape sequence as "A, n, o, A, n, o" and described the A, n and o letter shapes on the face of the model. In doing so, Dylan showed that he recognised embedded shapes (shapes within shapes) and embedded patterns (pattern within a pattern). Dylan identified these shapes in his model and was then able to identify another pattern as "space, yellow, space, yellow", or "orange yellow, orange yellow" by looking at his model from a variety of perspectives. This showed his use of transformation skills to find the pattern repetition. Figure 8 depicts his attempt to show three-dimensionality in the drawing by showing that the structure is not flat. This is unusual and impressive at this age because he uses oblique lines to show depth.

Implications for pedagogy, curriculum and assessment

The wide variety and complexity of responses to this task allowed us to see the potential of the students and the possibilities that an open-ended approach can offer. All of the students were engaged in sustained investigation and independent mathematical thinking. Their understanding and application of a unit of repeat (as

a pattern) and their three-dimensional spatial/geometric concepts were made more transparent than if they had completed simple linear AB repetitions using blocks or symbols. The teacher was able to challenge the students further based on their level of integrated conceptual knowledge.

In terms of curriculum requirements, the students were engaged in a range of mathematics learning outcomes related to measurement, space and geometry, number and pattern, as well as reasoning about, and justification of, their responses. This open-ended but simple task allowed the teacher to glean much information about students' mathematical knowledge in a short time. It proved to be an effective way to cater for the varying needs of students in the classroom, extending individuals at their own pace and level. It was also possible to capitalise on the rich opportunities for engagement in visual and practical mathematical processes to develop students' expressive language.

The proposed Australian Curriculum (2009) recommends the use of open-ended, rich tasks to stimulate the development of mathematical proficiencies. Open-ended tasks are also recognised within the NSW Quality Teaching Model as providing high intellectual quality as students engage in higher-order thinking and opportunities for explanation of concepts explored (NSW DET, 2003).

Concluding comments

To use a colloquialism, teachers and learners can “work smarter not harder” by posing well formulated open-ended tasks that integrate a range of mathematical concepts in an efficient way. The formulation of the task presented in this paper had an explicit purpose but was implemented in such a way as to promote individuals’ conceptual ideas. It was built on practical ideas from research with young children.

The development of patterning and space/geometry concepts and multiplicative skills through this open-ended task is only one way to promote children’s mathematical creativity. Increasingly, new research may require us to raise expectations of young children’s abilities and at the very least, urge us to extend children beyond traditional, single focus programs.

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